Period 4, May 6, 2025

To save money for a sabbatical to earn a master's degree, Henry deposits \$2500 at the end of each year in an annuity that pays 6.7% compounded annually. Use the formula for the value of an annuity, shown to the right.

$$A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$$

a. How much will he have saved at the end of three years?
b. Find the interest.

$P = 2500$ $\quad r = 0.067$ $\quad n = 1$ $\quad T = 3$

a. How much money will be in the annuity three years later?

\$ 8014

(Do not round until the final answer. Then round to the nearest dollar as needed.)

b. How much interest will the annuity have gained?

\$

(Do not round until the final answer. Then round to the nearest dollar as needed.)

$$A = \frac{2500\left[\left(1+\frac{0.067}{1}\right)^{1\cdot 3} - 1\right]}{\frac{0.067}{1}}$$

$$\frac{2500\left[(1.067)^3 - 1\right]}{0.067} = \frac{2500[0.2147677]}{0.067}$$

$A = 8013.7201$

Total investment

$3 \cdot 2500 = 7500$

interest gained

$8014 - 7500 = 514$

$A = 800{,}000$

If a person at the age of 35 years decides to retire at age 55 with a retirement fund of \$800,000, how much should he deposit at the end of each month for the next 20 years in an IRA paying 10% annual interest compounded monthly to achieve his goal? Round to the nearest dollar.

$n = 12$ $\quad T = 20$ $\quad r = 0.10$

The formula for monthly deposit is $P = \dfrac{A \cdot r}{n\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}$.

$$P = \frac{800{,}000(0.10)}{12\left[\left(1+\frac{0.10}{12}\right)^{12\cdot 20} - 1\right]} = \frac{80{,}000}{12\left[(1.0083)^{240} - 1\right]} = \frac{80{,}000}{12(6.32807)}$$

The person should deposit \$ 1054 at the end of each month for the next 20 years.
(Round up to the nearest dollar.)

$P = 1053.5064$

≈ 1054

$$\frac{r}{n} A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\cancel{\frac{r}{n}}} \cdot \frac{r}{n} \Rightarrow \frac{Ar}{n} = P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right] \Rightarrow \frac{Ar}{n\left[\left(1+\frac{r}{n}\right)^{nT} - 1\right]}$$

A pendulum swings through an arc of 8 inches. On each successive swing, the length of the arc is 86% of the previous length. After 20 swings, what is the total length of the distance the pendulum has swung?

8,	0.86(8),	$0.86^2(8)$,	$0.86^3(8)$,	...
1st swing	2nd swing	3rd swing	4th swing	

$n=20$

The pendulum has swung through a distance of ☐ inches.
(Do not round until the final answer. Then round to the nearest hundredth as needed.)

$8+0.86(8)+(0.86)^2(8)+(0.86)^3(8)\ldots+(0.86)^{19}(8)$

geometric series

$a_1=8$

$r=0.86$

$n=20$

$S=\dfrac{a_1(1-r^n)}{1-r}$

$\dfrac{8(1-0.86^{20})}{1-0.86}=\dfrac{8(0.95102561)}{0.14}$

$=54.344$ in

$P=17{,}000$

To offer scholarship funds to children of employees, a company invests \$17,000 at the end of every three months in an annuity that pays 12.9% compounded quarterly. Use the formula for the value of an annuity.

$$A=\frac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$$

$r=0.129$ $n=4$

a. How much will the company have in scholarship funds at the end of fifteen years?
b. Find the interest.

$t=15$

a. How much money will be in the fund after fifteen years?

\$☐
(Do not round until the final answer. Then round to the nearest dollar as needed.)

$A=\dfrac{17{,}000\left[\left(1+\frac{0.129}{4}\right)^{4\cdot 15}-1\right]}{\frac{0.129}{4}}$

$\dfrac{17{,}000\,[6.715734-1]}{0.03225}$

3012945.109

invested $=(17{,}000)\cdot 4\cdot 15=1{,}020{,}000$

interest $3012945.109-1020000=1{,}992{,}945.109$

Find the sum of the infinite geometric series.

$$\sum_{i=1}^{\infty} 5(0.9)^{i-1} = 5(0.9)^{1-1} + 5(0.9)^{2-1} + 5(0.9)^{3-1} + 5(0.9)^{4-1} \ldots$$

$$5(0.9)^0 + 5(0.9)^1 + 5(0.9)^2 + 5(0.9)^3$$

geometric infinite series

$$S = \frac{a_1}{1-r} = \frac{5}{1-0.9} = \frac{5}{0.1} = 50$$

$$5 \cdot 1 + 5(0.9) + 5(0.81) + 5(0.729) + \ldots$$

$$\underbrace{5 + 4.5}_{0.9} \underbrace{+\,4.05}_{0.9} \underbrace{+\,3.645}_{0.9}$$

$a_1 = 5$
$r = 0.9$

Convert the equation to standard form by completing the square on x and y. Then graph the ellipse and give the location of its foci.

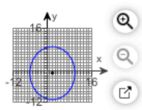
$64x^2 + 100y^2 - 256x + 200y - 6044 = 0$

The standard form of the equation is $\frac{(x-2)^2}{100} + \frac{(y+1)^2}{64} = 1$.

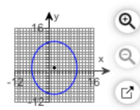
(Type an equation. Simplify your answer.)

Choose the correct graph below.

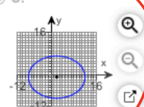
A.



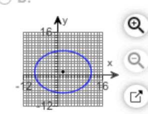
B.



C.



D.



The foci are located at $(-4,-1),(8,-1)$.

(Type ordered pairs. Use a comma to separate answers as needed. Simplify your answers. Type an exact answer, using radicals as needed.)

$$64x^2 - 256x \quad + 100y^2 + 200y \quad = 6044$$

$$64(x^2 - 4x + 4) + 100(y^2 + 2y + 1) = 6044 + 4 \cdot 64 + 1 \cdot 100$$

$6044 + 256 + 100$

$6300 + 100$

6400

$a = 1$
$b = -4$
$\frac{b}{2} = \frac{-4}{2} = -2$
$\left(\frac{b}{2}\right)^2 = (-2)^2 = 4$

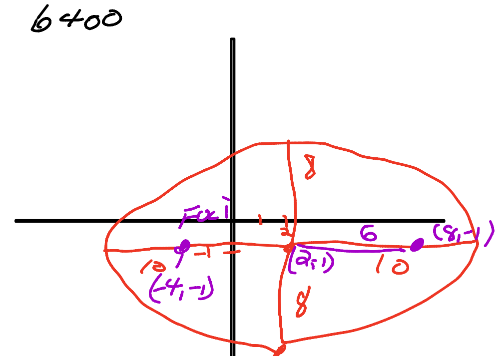
$a = 1$
$b = 2$
$\frac{b}{2} = \frac{2}{2} = 1$
$\left(\frac{b}{2}\right)^2 = (1)^2 = 1$

$$\frac{64(x-2)^2 + 100(y+1)^2}{6400} = \frac{6400}{6400}$$

$$\frac{64(x-2)^2}{6400} + \frac{100(y+1)^2}{6400} = 1$$

$$\frac{(x-2)^2}{100} + \frac{(y+1)^2}{64} = 1$$

center $(2,-1)$



$\text{Semiminor}^2 + \text{Foci}^2 = \text{semimajor}^2$

$8^2 + F^2 = 10^2$

$F = 6 = \text{Focal length}$

$$\left(x+\frac{b}{2}\right)^2 = \left(x+\frac{b}{2}\right)\left(x+\frac{b}{2}\right) = x^2 + \frac{bx}{2} + \frac{bx}{2} + \left(\frac{b}{2}\right)^2$$

$$x^2 + \frac{\cancel{2}bx}{\cancel{2}} + \left(\frac{b}{2}\right)^2$$

$$\left(x+\frac{b}{2}\right)^2 = x^2 + bx + \left(\frac{b}{2}\right)^2$$

$$\sqrt{\left(\frac{b}{2}\right)^2} = \sqrt{4}$$

$$\cancel{2} \cdot \frac{b}{\cancel{2}} = 2 \cdot 2$$

$$b = 4$$

$3, 3 \cdot r, 3 \cdot r^2 \ldots$

Find the sum of the infinite geometric series.

$$3 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \cdots \qquad S = \frac{a_1}{1-r} = \frac{3}{1-\frac{1}{5}}$$

$$\frac{\frac{3}{5}}{3} = \frac{3 \cdot r}{3}$$

$$\frac{1}{5} = r$$

$$\frac{\frac{3}{5^2}}{\frac{3}{5}} = \frac{\cancel{3}}{5^{\cancel{2}}} \cdot \frac{\cancel{5}}{\cancel{3}} = \frac{1}{5} = r$$

The sum of the infinite geometric series is $\boxed{\frac{15}{4}}$.

(Type an integer or a simplified fraction.)

$$\frac{3}{\frac{4}{5}} = \frac{3}{1} \cdot \frac{5}{4} = \frac{15}{4}$$